Projection

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Assume, $V = \mathbb{R}^3$, $B = \text{span}\{\overset{v_1}{(1,1,1)}, \overset{v_2}{(-1,-1,0)}\}$,

$C = \text{span}\{\overset{w_1}{(-1,0,1)}\}$. Clearly $V = B + C$

and also $V \cong B \oplus C$.

Find P_1 and P_2

$P_1 : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ (s.t. Range $P_1 = B$)

$P_1(av_1 + bv_2 + cw_1) = av_1 + bv_2$. Hence

$P_1(v_1) = v_1$, $P_1(v_2) = v_2$, $P_1(w_1) = 0_V$

$$M_{B,e} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = M \underbrace{\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}_{Q}.$$

So $M = M_{B,e} Q^{-1}$. ($M \to$ standard matrix for P_1)

$P_2 : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$P_2(av_1 + bv_2 + cw_1) = cw_1$. Hence $P_2(v_1) = 0_V$,

$P_2(v_2) = 0_V$, $P(w_1) = w_1$

so

$$M_{C,e} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Longrightarrow$$

$$M Q = M_{c,e}$$

$$M \underbrace{\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}_{Q} = \underbrace{\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{M_{c,e}} \Rightarrow$$

$M = M_{c,e} Q^{-1}$. (standard matrix for B_2)

Assume $T: \mathbb{R}^3 \to \mathbb{R}^3$ s.t.
$3, -2$ are eigenvalues of T, and
$E_3 = B$, $E_{-2} = C$. (B, C as before).

$B_1(T): \mathbb{R}^3 \to \mathbb{R}^3$

$B_1(T)(av_1 + bv_2 + cw_1) = T(av_1 + bv_2)$.

<u>Now (Math result):</u>

standard matrix rep. of $B_1(T)$
$= 3 \times$ standard matrix rep. of B_1

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same thing: standard matrix rep. of $B_2(T): \mathbb{R}^3 \to \mathbb{R}^3$ s.t. $B_2(T)(av_1 + bv_2 + cw_1)$
$= T(cw_1)$ is $-2 \times$ standard matrix rep. of B_2.

Find Smith form of

$$A = \begin{bmatrix} 2 & 6 & 4 \\ -2 & -4 & 8 \\ 0 & 0 & 6 \end{bmatrix} \text{ over } \mathbb{Z}$$

1st $|A| = 24$.

so we need R, C invertible over $\mathbb{Z}$ s.t. $RAC = \underbrace{\begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}}_{D}$ s.t.

$d_1 \mid d_2 \mid d_3$ and $|D| = d_1 d_2 d_3 = \pm |A|$.

steps.

① stare at A. gcd(all numbers in A) = 2

So $d_1 = \pm 2$. Since $d_1 d_2 d_3 = \pm 24$ and $d_1 = \pm 2$ and $d_1 | d_2 | d_3$, we conclude $d_2 = \pm 2$, $d_3 = \pm 6$.

See work

$\Longrightarrow$

We get R from I_3 (because A is 3×3) by using row operations (only, $\alpha R_i + R_k \to R_k$, $R_i \leftrightarrow R_k$)

We get C from I_3 by using columns operations (only, $\alpha C_i + C_k \to C_k$ and $C_i \leftrightarrow C_k$).

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 4 \\ -2 & -4 & 8 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 + R_2 \to R_2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 6 & 4 \\ 0 & 2 & 12 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$-3C_1 + C_2 \to C_2$$
$$-2C_1 + C_3 \to C_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 12 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$-6C_2 + C_3 \to C_3$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 6 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 1 & -3 & 16 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}}_{C}$$

Hence $RAC = D$

check!